Engineering Analysis with Boundary Elements 34 (2010) 549–556

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

An *hpr*-mesh refinement algorithm for BEM

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ARTICLE INFO

Article history:
Received 28 October 2009
Accepted 6 January 2010
Available online 18 February 2010

Keywords:
Boundary element method
Mesh refinement

ABSTRACT

This paper describes a mesh refinement scheme for boundary element method in which the number of elements, the size of elements, element end locations and the element polynomial order are determined to meet the user specified accuracy. The use of grading function in conjunction with the L_1 norm makes the mesh refinement scheme applicable to a variety of boundary element formulation and applications. The algorithm is stable for smooth, discontinuous, as well as singular density functions. Numerical results for mathematical test functions as well as for elastostatic problems demonstrate the viability and versatility of the algorithm for BEM applications.

1. Introduction

Computational tools like finite element method (FEM) and boundary element method (BEM) have become integral part of engineering design. Efforts to improve the accuracy of the analysis by refining the mesh have been going on since late seventies [1]. The mesh refinement schemes in BEM, like in FEM, can be classified [2] into the *h*, *p*, and *r* refinement and their combination. In the *h*-method, the accuracy is improved by increasing the total number of elements, but the order of the polynomial remains invariant. The high computational cost associated with the *h*-method due to the large number of unknowns can be reduced to some extent by the *h*-hierarchical refinement scheme [3]. In the *p*-method the polynomial order is increased uniformly or selectively to improve the accuracy while keeping the node location and element length unchanged during the iterative process [4]. The convergence rate of *p*-method is always better than the *h*-method for smooth functions [5] but may not converge near a singularity [6]. In the *r*-method [7], the total number of elements and the order of the polynomial are kept invariant, but the spacing of the elements is adjusted to minimize the error. If the initial mesh does not have sufficient degrees of freedom (DOF) then the desired accuracy cannot be obtained with the *r*-method [8].

The *hp*-method is a combination of the *h* and *p*-method that overcomes the convergence difficulty of the *p*-method near the singularity. Initial papers published on *p* and *hp* mesh refinement method were mostly by Babuska and his colleagues [9]. Most of Babuska work was done in relation to FEM and later extended to BEM [10]. In FEM he showed that mesh generated using grading function performs better than a graded mesh [9]. In FEM he has shown that optimal meshes are graded towards the singular point and starting with the second element away from singularity the polynomial order increases linearly. He has shown that for the Galerkin BEM, *hp*-method, with geometric mesh graded towards the point of singularity, has exponential rate of convergence [11]. In some cases the location [5] and the strength of the singularity [10] has to be specified. The *p*-method and the *hp*-method are usually restricted to the Galerkin method of satisfying the boundary conditions in BEM due to the difficulty of determining the collocation points with the increase in polynomial order.

Sun and Zamani [8] developed a *hr*-method for direct BEM. Finding residuals is computationally expensive in case of indirect BEM and thus the use of residuals as an error indicator makes the method suitable primarily for the direct BEM. Ammons and Vable [12] developed a *hr*-method which can be used for both direct and indirect BEM by using grading function [7] and L_1 norm. The *hr*-method of Ammons and Vable has very slow convergence rate near discontinuities and singularities in density function and can have very large DOF if a lower order polynomial is specified by the user.

This paper presents an *hpr*-method that has fast convergence for non-singular density functions and is stable for discontinuous and singular density functions. The use of grading function with L_1 norm as an error measure makes it possible to use the refinement scheme for approximation of any mathematical function of one variable and thus is independent of BEM methodology and its application. The three functions used for identification of critical ideas and for testing the algorithm are:

$$\text{Function 1}: \quad u=\sin(\pi s) \quad 0\le s\le 1 \tag{1a}$$

$$\text{Function 2}: \quad u=\sqrt{s} \quad 0\le s\le 1 \tag{1b}$$

$$\text{Function 3}: \quad u=1/\sqrt{s} \quad 0\le s\le 1 \tag{1c}$$

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doi:10.1016/j.enganabound.2010.01.004